

# Lecture 09/21/23 Systems of Linear Equations

HW Due: 16      17      18  
                 Fri      Sun      Wend

No quiz!

Defn: A system of linear equations is a set of 2 or more linear equations consisting of the same types of variables.

eg

$$\begin{cases} 2x + y = 2 \\ 4x - 2y = 0 \end{cases}$$

There may be more than two variables!

Defn: a solution to a system of linear equations is an pair of numbers  $(a, b) \leftrightarrow (x, y)$  that solve ~~both~~ all equations in the system at the same time. It is possible to have no sols!

Prob 1: i)  $\begin{cases} 5x - 3y = -12 \\ 2x + 3y = -9 \end{cases}$   $(-3, -1)$  sol

$$5(-3) - 3(-1) \stackrel{?}{=} -12$$

$$-15 + 3 = -12 \quad \checkmark$$

$$2(-3) + 3(-1) \stackrel{?}{=} -9$$

$$-6 + -3 = -9 \quad \checkmark$$

Yes it is a sol.

ii)

$$\begin{cases} 3x - y = -8 \\ 2x + 3y = 5 \end{cases}$$

$$3(-3) - (-1) \stackrel{?}{=} -8$$

$$2(-3) + 3(-1) \stackrel{?}{=} 5$$

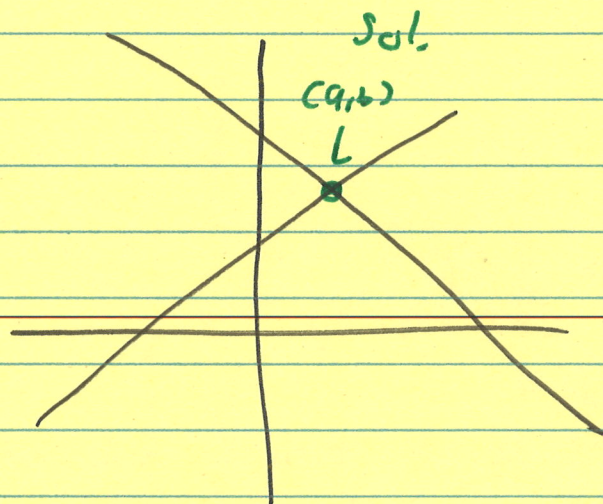
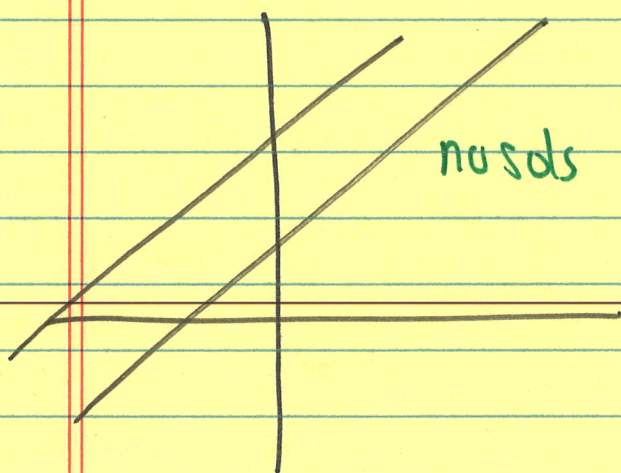
$$-9 + 1 = -8 \quad \checkmark$$

$$-6 + -3 = -9 \neq 5$$

Not a Sol.

Substitution

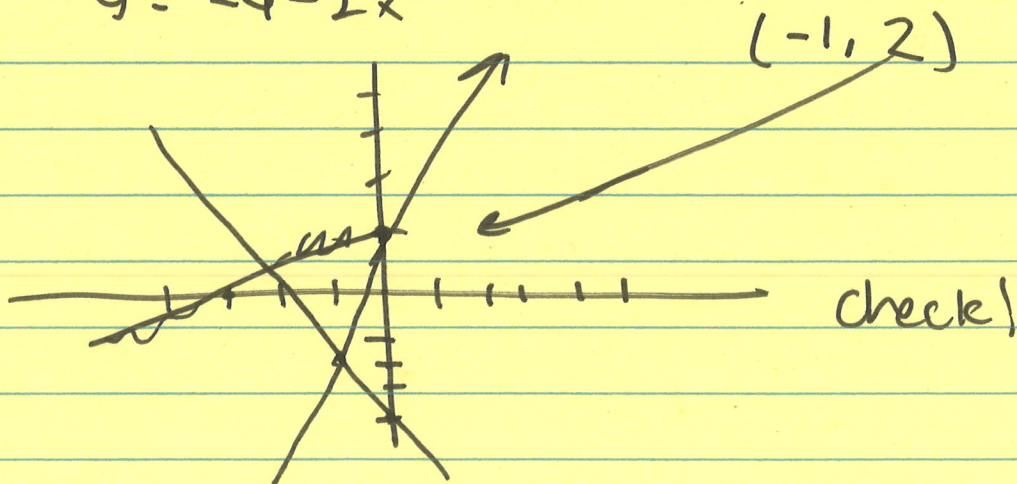
Graphing: The solution to a system of equations is their point of intersection



2a) ① ~~Graph~~ S-IF

$$\left. \begin{array}{l} \rightarrow \\ \rightarrow \end{array} \right\} \begin{array}{l} y = 1 + 3x \\ y = -4 - 2x \end{array}$$

② Graph



# Substitution

Ex: 
$$\begin{cases} 2x + 2y = 16 \\ y = 2x - 5 \end{cases} \quad \begin{cases} 2x - 5y = 11 \\ 3x = 2y \end{cases}$$

① Solve one equation for some variable

Choose an easy one to solve.

~~y =~~  $x = \frac{2}{3}y$

$y = \frac{3}{2}x$  ← Another option

② Substitute into equation not used in ①

~~$2x - 5y = 11$~~

~~$2x - 5\left(\frac{2}{3}x\right) = 11$~~

~~$2x - \frac{10}{3}x = 11$~~

~~$-\frac{4}{3}x = 11$~~

~~$x = \frac{-33}{4}$~~

$2\left(\frac{2}{3}y\right) - 5y = 11$

$\frac{4}{3}y - 5y = 11$

$-\frac{11}{3}y = 11$

$y = \frac{-33}{11}$

③ solve for y when  ~~$x = \frac{2}{3}$~~  with sol in ①

$x = \frac{2}{3}y$

~~$x = \frac{2}{3}\left(\frac{11}{6}\right) = \frac{11}{9}$~~

$x = \frac{2}{3}(-3)$

$= -2$

~~y =~~

~~$x = \frac{2}{3}y$~~

~~$x = \frac{2}{3}\left(\frac{11}{6}\right) = \frac{11}{9}$~~

~~$x = \frac{2}{3}(-3)$~~

~~$= -2$~~

from ①

~~$x = \frac{2}{3}\left(\frac{11}{6}\right) = \frac{11}{9}$~~

~~$x = \frac{2}{3}(-3)$~~

~~$= -2$~~

$$-\frac{33}{5} + \frac{-99}{32}$$

4) write as point  $(\frac{-33}{5}, \frac{-99}{32})$

$(\frac{-33}{4}, \frac{-99}{8})$

$(\frac{-21}{3}, \frac{-1}{3})$

$(-2, -3)$

Check!

Elimination:

$$\begin{cases} y - x = 3 \\ 3y + 6x = -9 \end{cases}$$

① Multiply equations by constants so that when added a variable is eliminated

$$-3(y - x) = -3 \cdot 3$$

$$\begin{cases} -3y + 3x = -9 \\ 3y + 6x = -9 \end{cases}$$

Does not  
change  
system!  
sol.

② Add equations

$$9x = -18$$

$$x = -2$$

③ Solve for remaining var. using <sup>an</sup> equation.

$$y - x = 3$$

$$y - 2 = 3$$

$$y = 1$$

$$(-2, 1)$$

check!